Name of College - S.S. College, J-Bad

Deptt - Mathematics

PAGE NO.: 1

TOPIC - St-line (Problem)

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Analytical Geometry of 3-D

Class - B.Sc I (Sub + Hons)

Time — 1:00 P.M to 1:45 PM

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Study Material

Coplanar lines → Condition under which two Given st. lines should be co-planar.

Let the equation of two given st. lines be

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n} \quad \text{—— (I)}$$

and

$$\frac{x-\alpha_1}{l_1} = \frac{y-\beta_1}{m_1} = \frac{z-\gamma}{n_1} \quad \text{—— (II)}$$

the equation of any plane passing through 1st line is

{ Since equation of plane passing through (x_1, y_1, z_1) is Given by

$$a(x-x_1) + b(y-y_1) + c(z-z_1) = 0$$

(ii) If $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ be the equation

→ st. line passing through (x_1, y_1, z_1) and l, m, n are direction cosines of a st. line parallel to it }

Now Equation of any plane passing through the First line is

$$a(x-\alpha) + b(y-\beta) + c(z-\gamma) = 0 \quad \text{—— (III)}$$

Where $al + bm + cn = 0$ —— (IV)

Since plane (iii) will contain the st. line (ii) if the point $(\alpha_1, \beta_1, \gamma_1)$ lie on it and the line is perpendicular to the normal to it.

The condition for this is

$$a(\alpha-\alpha_1)+b(\beta-\beta_1)+c(\gamma-\gamma_1)=0 \quad (5)$$

and $al_1 + bm_1 + cn_1 = 0$

Now eliminating a, b, c between

$$al+bm+cn=0$$
$$a(\alpha-\alpha_1)+b(\beta-\beta_1)+c(\gamma-\gamma_1)=0$$
$$al_1+bm_1+cn_1=0$$

we have

$$\begin{vmatrix} l & m & n \\ \alpha-\alpha_1 & \beta-\beta_1 & \gamma-\gamma_1 \\ l_1 & m_1 & n_1 \end{vmatrix}=0$$

Which is the required condition.

Also elimination a, b, c between (iii) (iv) and (6) we get.

$$\begin{vmatrix} x-\alpha & y-\beta & z-\gamma \\ l & m & n \\ l_1 & m_1 & n_1 \end{vmatrix}=0$$

This is the required ~~[illegible]~~ equation of

Plane containing given two st.-lines

Ex:→ Prove that the lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

and $$\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$$

are coplanar. Find their point of intersection and the plane in which they lie.

Solution → Given lines are

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \quad \text{——— (i)}$$

and $$\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5} \quad \text{——— (ii)}$$

Since 1st passes through (1, 2, 3)
and direction ratios are 2, 3, 4

2nd passes through (2, 3, 4)
and direction cosines are (3, 4, 5)

Since we know two lines

$$\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$$

and $$\frac{x-\alpha_1}{l_1} = \frac{y-\beta_1}{m_1} = \frac{z-\gamma_1}{n_1}$$

are coplanar if $$\begin{vmatrix} \alpha-\alpha_1 & \beta-\beta_1 & \gamma-\gamma_1 \\ l & m & n \\ l_1 & m_1 & n_1 \end{vmatrix} = 0$$

Here $\alpha-\alpha_1 = 1-2 = -1$ $\quad l = 2 \quad m = 3 \quad n = 4$

$\beta-\beta_1 = 2-3 = -1$ $\quad l_1 = 3 \quad m_1 = 4 \quad n_1 = 5$

$\gamma-\gamma_1 = 3-4 = -1$

Now $\begin{vmatrix} \alpha-\alpha_1 & \beta-\beta_1 & \gamma-\gamma_1 \\ l & m & n \\ l_1 & m_1 & n_1 \end{vmatrix}$

$= \begin{vmatrix} -1 & -1 & -1 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$

$= -1 \begin{vmatrix} 3 & 4 \\ 4 & 5 \end{vmatrix} + \begin{vmatrix} 2 & 4 \\ 3 & 5 \end{vmatrix} - 1 \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix}$

$= -(15-16) + (10-12) - (8-9)$

$= 1 - 2 + 1 = 0$

Thus given two lines lie in the same plane.

For point of intersection

say $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$

$\Rightarrow x = 2\lambda+1$

$y = 3\lambda+2$

$z = 4\lambda+3$

Also say

$$\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5} = k$$

$\Rightarrow x = 3k+2, \; y = 4k+3, \; z = 5k+4$

If these two lines intersect, the co-ordinate of P will satisfy both (i) and (ii) therefore, we have

$2\lambda+1 = 3k+2 \Rightarrow 3k - 2\lambda = -1 \Rightarrow 9k - 6\lambda = -3$

$3\lambda+2 = 4k+3 \Rightarrow 4k - 3\lambda = -1 \quad \underline{\overset{-}{}8k \overset{+}{-} 6\lambda = \overset{+}{-}2}$

$4\lambda+3 = 5k+4 \Rightarrow 5k - 4\lambda = -1 \quad k = -1$

put $k = -1$

$\therefore -3 - 2\lambda = -1$

$-2\lambda = 2$

$\lambda = -1$

Thus point of intersection of these two st-line is $(-1, -1, -1)$

Problem Find the equation of the plane passing through $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ and perpendicular to the plane containing

$$\frac{x}{m} = \frac{y}{n} = \frac{z}{l} \text{ and}$$

$$\frac{x}{n} = \frac{y}{l} = \frac{z}{m}$$

Solution $\rightarrow$ Let the equation of required plane be

$$ax + by + cz + d = 0$$

Since the plane contain the line,

$$\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$$

$\Rightarrow$ it passes through the origin $(0,0,0)$

$\Rightarrow d = 0$

DATE: 14-08-2020

Now Equation of the plane becomes $ax+by+cz=0$

Since the above line lies in

$$ax+by+cz=0 \quad \text{--- (1)}$$

therefore $al+bm+cn=0$ --- (2)

Now the equation of any line passing through through the origin and perpendicular to (1) is

$$x/a = y/b = z/c$$

Let the plane containing the last two lines be denoted by π.

Since the plane (1) is perpendicular to π, therefore the line

$x/a = y/b = z/c$ lies in π

the three lines

$$x/a = y/b = z/c$$

$$x/m = y/n = z/l$$

and $x/n = y/l = z/m$ are coplanar.

the condition for this is

$$\begin{vmatrix} a & b & c \\ m & n & l \\ n & l & m \end{vmatrix} = 0$$

i.e $a(mn-l^2) - b(m^2-nl) + c(ml-n^2) = 0$

$\Rightarrow a(mn-l^2) + b(nl-m^2) + c(ml-n^2) = 0$ --- (3)

Solving (C) and (B)

by Cross-multiplication, we get-

$$\frac{a}{m(lm-n^2)-n(nl-m^2)} = \frac{b}{n(mn-l^2)-l(lm-n^2)} = \frac{c}{l(nl-m^2)-m(mn-l^2)}$$

$$\Rightarrow \frac{a}{(m-n)(ml+mn+nl)} = \frac{b}{(n-l)(lm+mn+nl)} = \frac{c}{(l-m)(lm+mn+nl)}$$

$$\Rightarrow \frac{a}{m-n} = \frac{b}{n-l} = \frac{c}{l-m}$$

Putting these value of a, b, c in

$ax+by+cz=0$.

$(m-n)x+(n-l)y+(l-m)z=0$